

Progression in Calculation



Aims

The national curriculum for mathematics aims to ensure that all pupils:

- become **fluent** in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.
- **reason mathematically** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
- can **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

Introduction

Written methods of calculations are based on mental strategies. Each of the four operations builds on mental skills which provide the foundation for jottings and informal written methods of recording. Skills need to be taught, practised and reviewed constantly. These skills lead on to more formal written methods of calculation.

Strategies for calculation need to be represented by models and images to support, develop and secure understanding. This, in turn, builds fluency. When teaching a new strategy it is important to start with numbers that the child can easily manipulate so that they can understand the methodology.

The transition between stages should not be hurried as not all children will be ready to move on to the next stage at the same time, therefore the progression in this document is outlined in stages. Previous stages may need to be revisited to consolidate understanding when introducing a new strategy.

A sound understanding of the number system is essential for children to carry out calculations efficiently and accurately.

Magnitude of Calculations

Year 1 – $U + U$, $U + TU$ (numbers up to 20) including adding zero, $U - U$, $TU - U$ (numbers up to 20) including subtracting zero, $U \times U$, $U \div U$

Year 2 - $TU + U$, $TU +$ multiples of 10, $TU + TU$, $U + U + U$, $TU - U$, $TU -$ tens, $TU - TU$, $TU \times U$, $U \div U$

Year 3 – add numbers with up to three-digits, $HTU +$ multiples of 10, $HTU +$ multiples of 100, subtract numbers up to three-digits, $HTU - U$, $HTU -$ multiples of 10, $HTU -$ multiples of 100, $HTU - HTU$, $TU \times U$, $TU \div U$

Year 4 - add and subtract numbers with up to four-digits, $ThHTU + ThHTU$, $ThHTU - ThHTU$, add and subtract decimals with up to two decimal places in the context of money, multiply three numbers together, $TU \times U$, $HTU \times U$, $TU \times U$, multiply by zero and one, $TU \div U$, $HTU \div U$

Year 5 – add and subtract numbers with more than four-digits, add and subtract decimals with up to three decimal places, $ThHTU \times U$, $ThHTU \times TU$, $HTU \times TU$, multiply whole numbers and decimals with up to three-decimal places by 10, 100 and 1000, divide numbers with up to four-digits by U (including remainders as fractions and decimals and rounding according to the context)

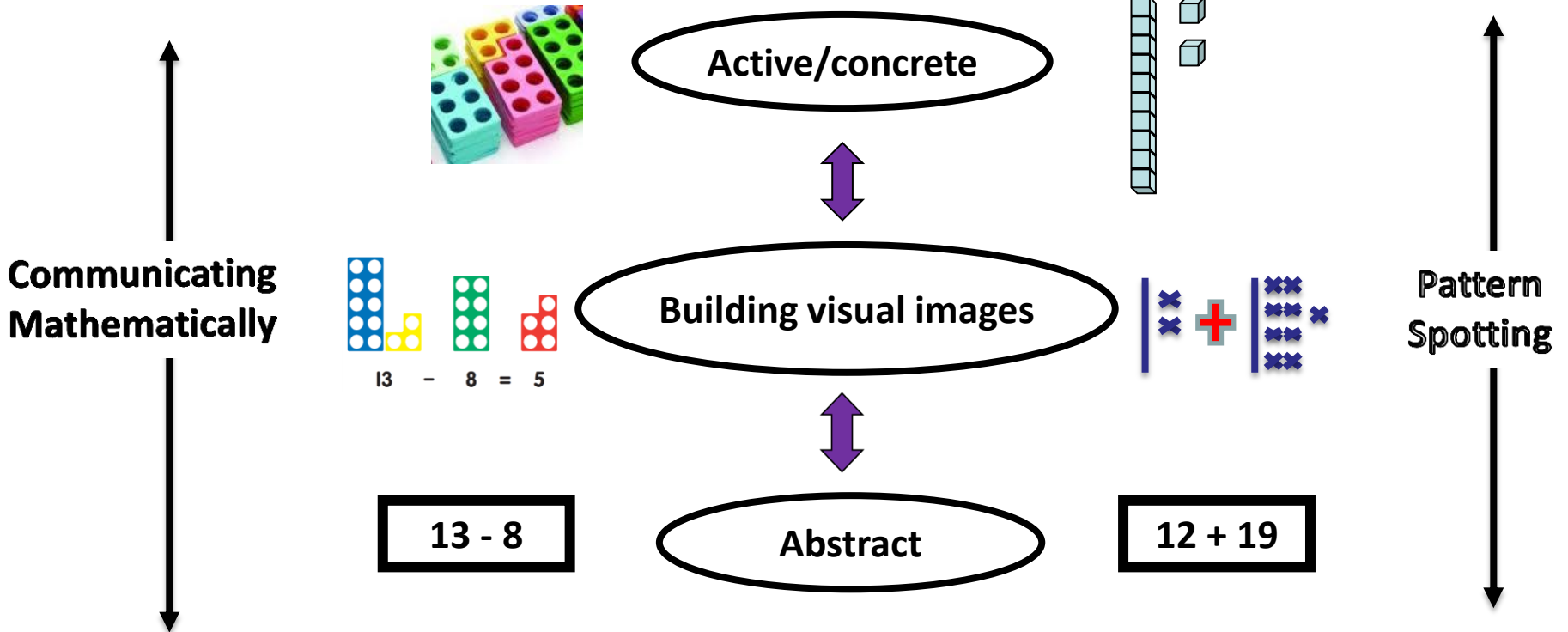
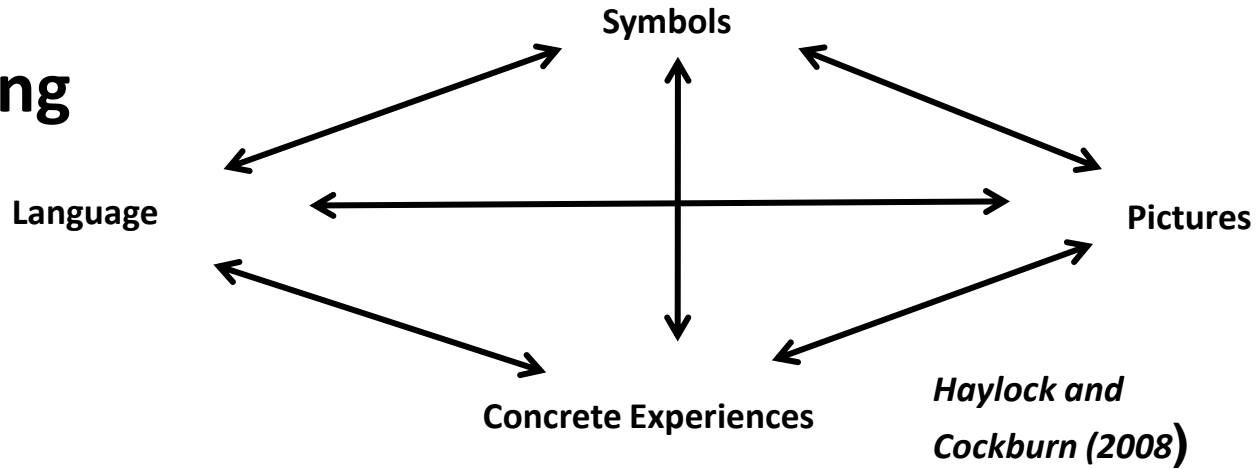
Year 6 - add and subtract numbers with more than four-digits, add and subtract decimals with up to three decimal places, multiply numbers with up to four-digits by TU , multiply numbers with up to two-decimal places by a whole number, divide numbers up to four-digits by TU (interpreting remainder according to the context), divide decimals up to two-decimal places by U or TU

Mathematics is an interconnected subject in which pupils need to be able to move fluently between representations of mathematical ideas. ... pupils should make rich connections across mathematical ideas to develop fluency, mathematical reasoning and competence in solving increasingly sophisticated problems. They should also apply their mathematical knowledge to science and other subjects.

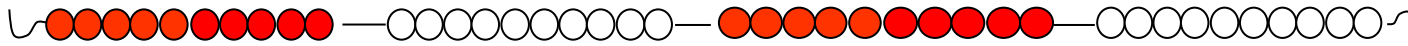
National Curriculum 2014

Structuring Learning

Children must have concrete experiences that enable them to create visual images. They should be encouraged to articulate their learning and to become pattern spotters.



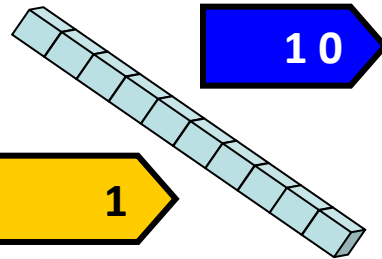
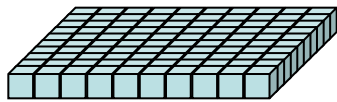
bead string



count stick

place value apparatus

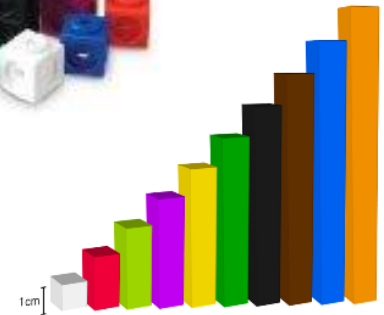
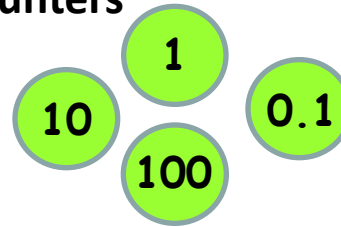
Hundreds	Tens	Units/Ones
100s	10s	1s



Multilink

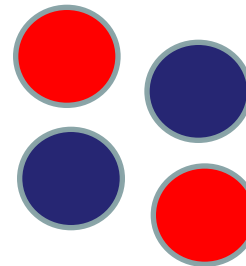
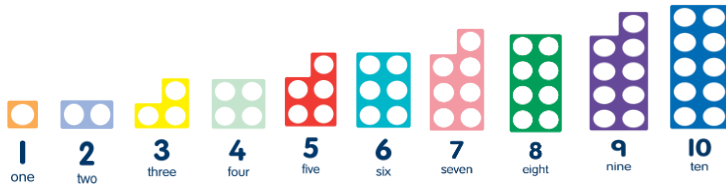


place value counters



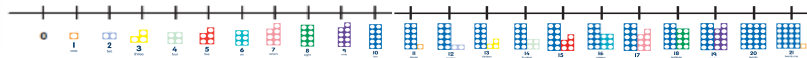
Cuisenaire

Numicon



double sided counters

number line



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140
141	142	143	144	145	146	147	148	149	150
151	152	153	154	155	156	157	158	159	160
161	162	163	164	165	166	167	168	169	170
171	172	173	174	175	176	177	178	179	180
181	182	183	184	185	186	187	188	189	190
191	192	193	194	195	196	197	198	199	200

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

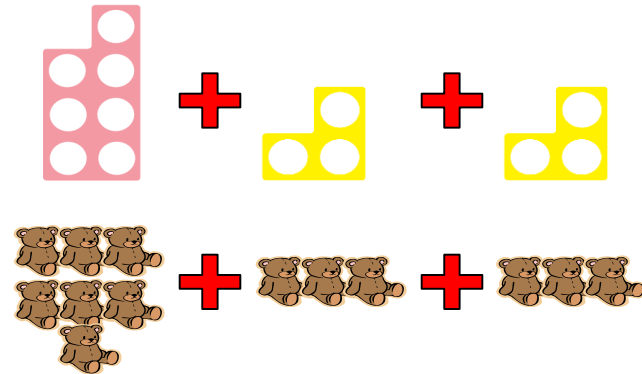
number grids
100 and 200

Structures of Addition (Haylock and Cockburn 2008)

Children should experience problems with all the different addition structures in a range of practical and relevant contexts e.g. money and measurement

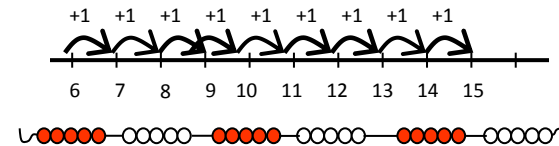
Aggregation

*Union of two sets
How many/much altogether?
The total*



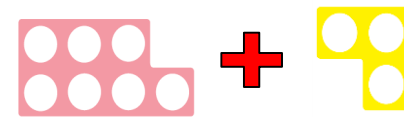
Augmentation

*Start at and count on
Increase by
Go up by*

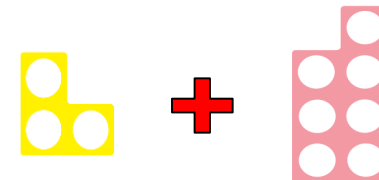


Commutative law

*Understand addition can be done in any order
Start with bigger number when counting on
(Explain to children that subtraction does not have this property)*



is the same as/equal to (=)



Addition

Pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly.
Addition and subtraction should be taught together.

End of Year Expectations

Possible Concrete and Visual Representations

Children's Recording

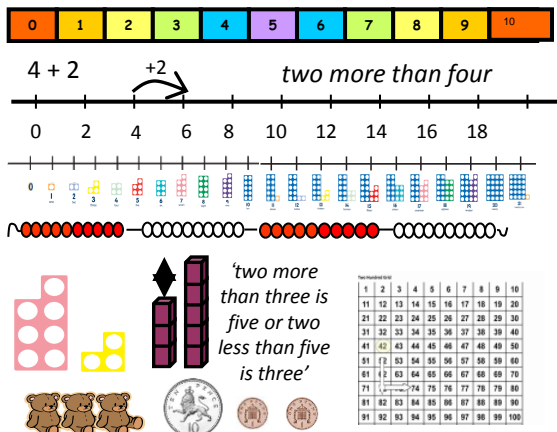
Fluency

Year 1

Children must experience combining two, **and then more than two**, groups of objects using counting on and the language of addition e.g. add, plus

Children must experience increasing numbers e.g. what is two more than seven?

Compare quantities to say how many less and/or how many more



If using Numicon, children could use printed Numicon icons and stick these in - progressing to recording number sentences alongside

$$1 + 2 = 3$$

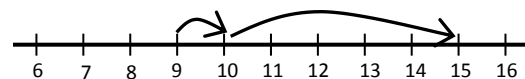
Children may record pictorially progressing to recording number sentences alongside

Example

9 and 6

$$9 + 6$$

$$9 + 6$$



Count forwards, to and across 100, beginning with 0 or 1 or from any given number

Switch count between tens and ones e.g. 10, 20, 30, 31, 32, 33 ...

Represent and use number bonds up to 20 (establish addition and subtraction as related operations)

Find one more than a number

Find ten more than a number

Count in multiples of 2s, 5s and 10s starting on multiples to highlight pattern recognition

Year 2

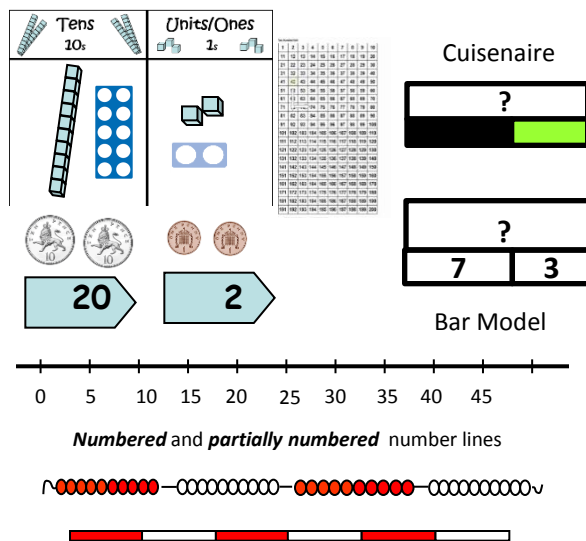
Children should be able to partition numbers in different ways e.g. as 2+2+2+1 or 5+3 or 23 as 20+3 or 10+13

Children should use concrete objects, pictorial representations and add numbers in different contexts e.g. money, measures

ENSURE CHILDREN HAVE THE OPPORTUNITY TO ADD **MORE** THAN TWO NUMBERS

Children should understand the language of sum

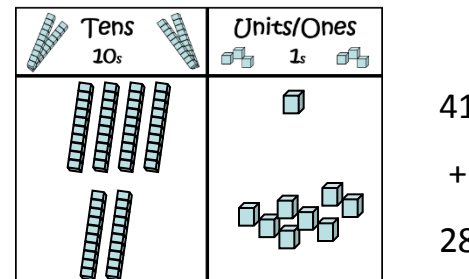
Ensure children understand that addition is commutative (can be done in any order)



Use Numicon, number grids, place value apparatus/Dienes, place value grids, place value cards, Encourage children to partition numbers rather than counting in ones.

Children apply, develop and secure their understanding of place value

Use jottings and record number sentences



$$40 + 20 = 60$$

$$1 + 8 = 9$$

$$60 + 9 = 69$$

Show increasing fluency in deriving pairs of numbers up to 10 and then up to 20

Use knowledge to derive and use number facts up to 100

Add numbers mentally including TU + U, TU + tens, TU + TU, U + U + U

End of Year Expectations

Year 3

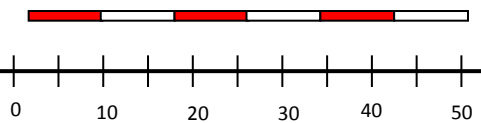
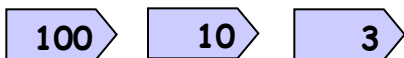
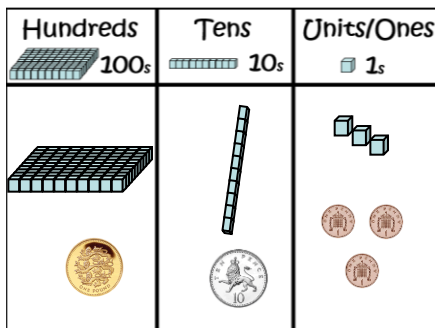
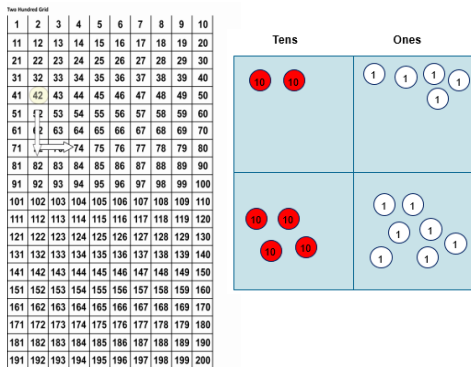
Add numbers with up to three-digits
(leading to formal written column method)

ENSURE CHILDREN HAVE THE OPPORTUNITY TO ADD **MORE THAN TWO NUMBERS** WITH DIFFERING NUMBERS OF DIGITS

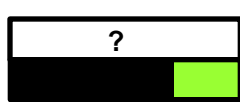
Children should partition numbers, up to 1000, in different ways
e.g. $100 + 40 + 6$ or $100 + 30 + 16$

Solve problems in different contexts including missing number problems

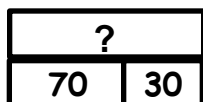
Possible Concrete and Visual Representations



Partially numbered and blank number lines



Cuisenaire



Bar Model

Teacher Modelling/Children's Recording

Children apply, develop and secure their understanding of place value and begin to record in columns

Manipulatives SHOULD be used alongside algorithms

Column addition (no exchanging) with up to three-digits

$$\begin{array}{r} 40 + 1 \\ + 20 + 8 \\ \hline 60 + 9 = 69 \end{array} \qquad \begin{array}{r} 40 + 3 \\ + 20 + 8 \\ \hline 70 + 1 = 71 \\ 10 \end{array}$$

Expanded recording without exchange

Expanded recording with exchange

$$\begin{array}{r} 100 + 40 + 1 \\ + 100 + 20 + 8 \\ \hline 200 + 60 + 9 = 269 \end{array}$$

Expanded recording

$$\begin{array}{r} \text{HTU} \\ 141 \\ + 128 \\ \hline 269 \end{array}$$

Compact (column) recording

$$\begin{array}{r} 143 \\ + 128 \\ \hline 271 \\ 1 \end{array}$$

Column addition (with exchanging)

$$\begin{array}{r} \text{HTU} \\ 789 \\ + 642 \\ \hline 1431 \\ 11 \end{array}$$

Compact (column) recording

$$\begin{array}{r} \pounds 7.89 \\ + \pounds 6.42 \\ \hline \pounds 14.31 \\ 11 \end{array}$$

Add decimals in the context of money

Fluency

Count in ones, tens and hundreds maintaining fluency through varied and frequent practice

Count from 0 in multiples of 4, 8, 50 and 100

Find 10 or 100 more than a number

Mentally add HTU + ones, HTU + tens, HTU + hundreds

Perform mental calculations with two-digit numbers, the answer could exceed 100

Year 4

Add numbers with up to four-digits (formal written column method) including numbers with up to two decimal places in the context of money

ENSURE CHILDREN HAVE THE OPPORTUNITY TO ADD **MORE THAN TWO NUMBERS** INCLUDING DECIMALS, WITH DIFFERING NUMBERS OF DIGITS

Solve two-step problems in different contexts including missing number problems

Count in 6s, 7s, 9s, 25s and 100s

Find 1000 more than a number

Perform mental calculations with increasingly large numbers to aid fluency

Addition

Pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly.
Addition and subtraction should be taught together.

End of Year Expectations

Year 5

Add numbers with more than four-digits and decimals up to three places

(formal written column method)

N.B. ENSURE CHILDREN HAVE THE OPPORTUNITY TO ADD **MORE THAN TWO NUMBERS** INCLUDING DECIMALS, WITH DIFFERING NUMBERS OF DIGITS

Solve multi-step problems selecting and justifying methods

Perform mental calculations with increasingly large numbers

Year 6

Add numbers with more than four-digits and decimals up to three places

(formal written column method)

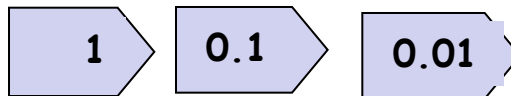
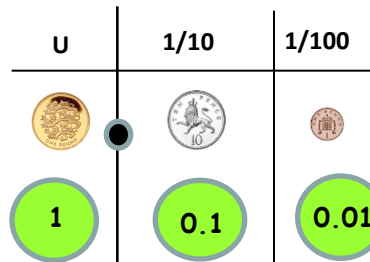
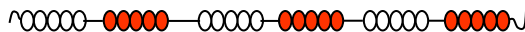
N.B. ENSURE CHILDREN HAVE THE OPPORTUNITY TO ADD **MORE THAN TWO NUMBERS**, INCLUDING DECIMALS, WITH DIFFERING NUMBERS OF DIGITS

Solve more complex calculations mentally

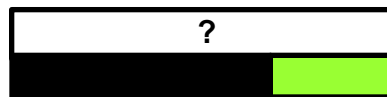
Solve multi-step problems in contexts, deciding which operations and methods to use and why

Possible Concrete and Visual Representations

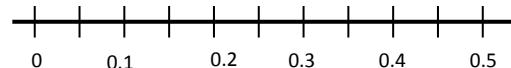
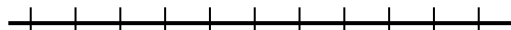
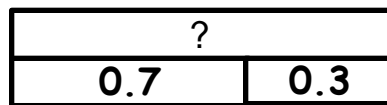
0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	2	3	4	5	6	7	8	9



Cuisenaire



Bar Model



Partially numbered and blank number lines



Teacher Modelling/Children's Recording

Manipulatives could be used alongside algorithms

$$\begin{array}{r} 2141 \\ + 1128 \\ \hline 3269 \end{array}$$

Column addition (no exchanging)

$$\begin{array}{r} 21.41 \\ + 1.12 \\ \hline 22.88 \end{array}$$

$$\begin{array}{r} 5189 \\ + 3128 \\ \hline 8317 \\ 11 \end{array}$$

Column addition (with exchanging)

$$\begin{array}{r} 51.89 \\ + 3.128 \\ \hline 55.018 \\ 11 \end{array}$$

Addition with decimals up to three decimal places including in different contexts e.g. money and measures

Fluency

Count forwards in powers of ten up to 100000

Count forwards in positive and negative whole numbers through zero

Practise mental calculations with increasingly large numbers

Practise fluency of written methods

Count in tens and hundreds increasing fluency of order and place value

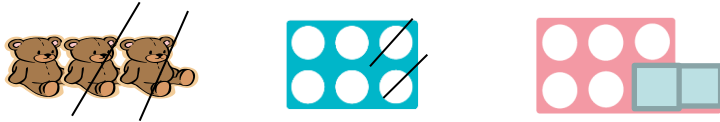
Perform increasingly complex mental calculations and those with increasingly large numbers to aid fluency

Structures of Subtraction (Haylock and Cockburn 2008)

Children should experience problems with all the different subtraction structures in a range of practical and relevant contexts e.g. money and measurement

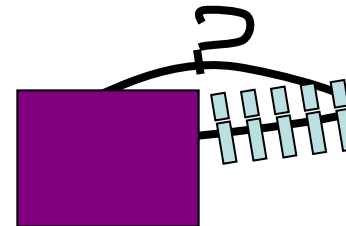
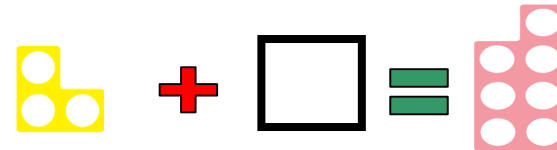
Partitioning

Take away
 ... how many left?
 How many are not?
 How many do not?



Inverse-of-addition

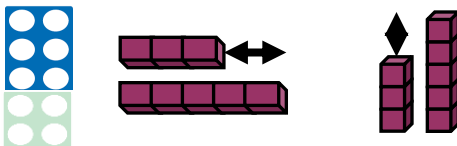
What must be added?
 How many (much) more needed?



There are ten pegs
 on the hanger –
 how many are covered?

Comparison

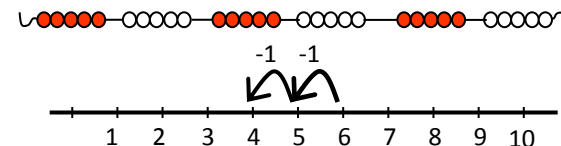
What is the difference?
 How many more?
 How many less (fewer)?
 How much greater?
 How much smaller?



'two more than three
 is five or two less than
 five is three'

Reduction

Start at and reduce by
 Count back by
 Go down by



Subtraction

Pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly.

Addition and subtraction should be taught together.

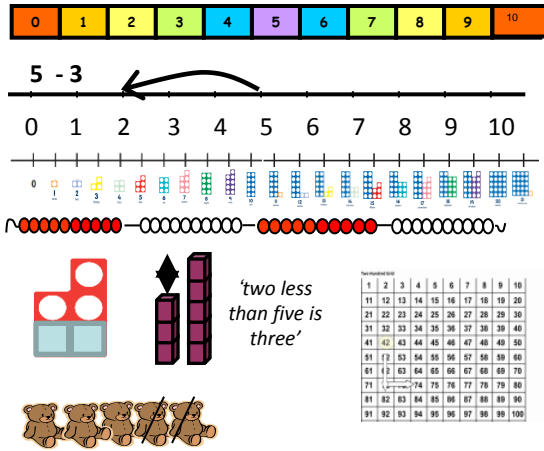
End of Year Expectations

Year 1

Understand subtraction as taking away
What is ... less than ...?

Compare quantities to say how many less and/or how many more

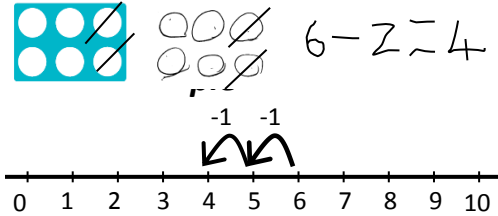
Possible Concrete and Visual Representations



Use practical resources such as bears, counters, cubes and number lines/hundred grids and progress to a resource such as Numicon to encourage counting back in groups rather than ones

Children's Recording

Children may begin recording pictorially progressing to recording number sentences alongside



Children could use printed Numicon icons and stick these in, again progressing to recording number sentences alongside

Fluency

Count backwards (including crossing 100) any given number

Switch count between ones and tens e.g. 33, 32, 31, 30, 20, 10

Represent and use subtraction facts linked to number bonds up to 20 (establish addition and subtraction as related operations)

Find one less than a number

Find ten less than a number

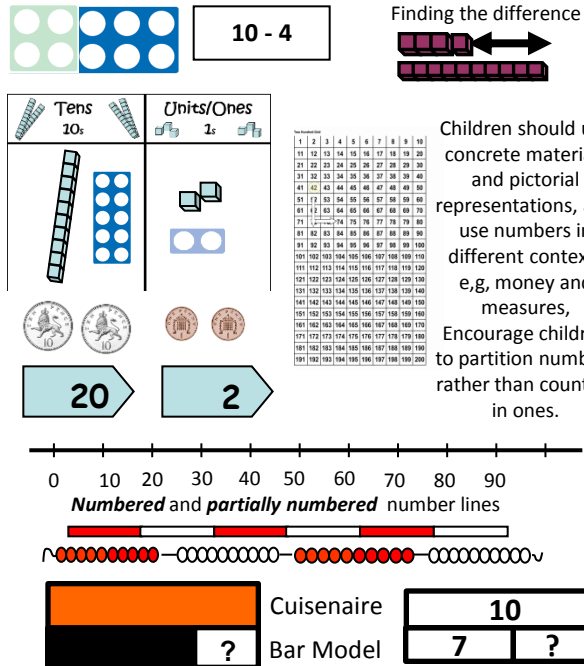
Count back in multiples of 2s, 5s and 10s starting on multiples to highlight pattern

Year 2

Understand subtraction as taking away and finding the difference

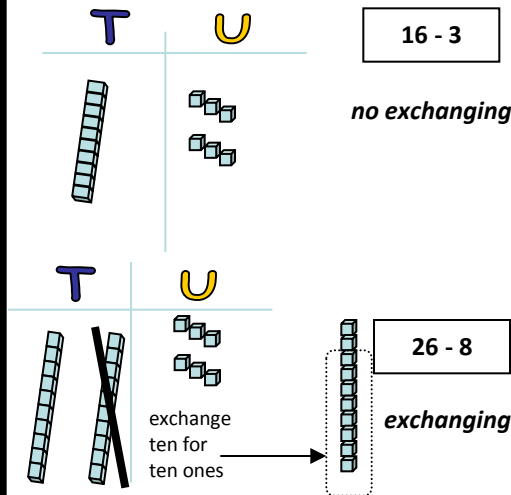
Ensure children understand that subtraction is not commutative (can not be done in any order)

Children should be able to partition numbers in different ways



Children should use concrete materials and pictorial representations, and use numbers in different contexts e.g. money and measures. Encourage children to partition numbers rather than counting in ones.

Children apply, develop and secure their understanding of place value and begin to record using jottings and number sentences



Practise addition and subtraction facts to 20

Show increasing fluency in deriving subtraction facts for numbers up to 10 and then up to 20

Use known facts to 20 to derive new facts e.g. $3 + 7 = 30 + 70$

Use knowledge to derive and use subtraction number facts up to 100

Subtraction

Pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly. Addition and subtraction should be taught together.

End of Year Expectations

Possible Concrete and Visual Representations

Teacher Modelling/ Children's Recording

Fluency

Year 3

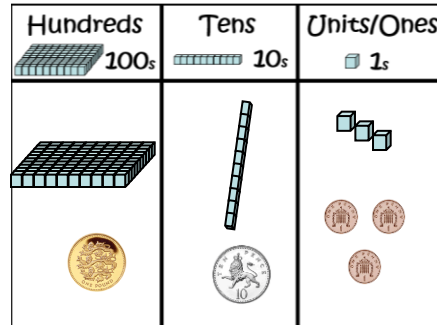
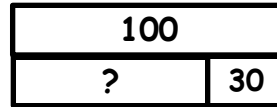
Subtract numbers with up to three-digits
(formal written column method)
Children apply, develop and secure their understanding of place value and begin to record in columns

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
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61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140
141	142	143	144	145	146	147	148	149	150
151	152	153	154	155	156	157	158	159	160
161	162	163	164	165	166	167	168	169	170
171	172	173	174	175	176	177	178	179	180
181	182	183	184	185	186	187	188	189	190
191	192	193	194	195	196	197	198	199	200

Cuisenaire

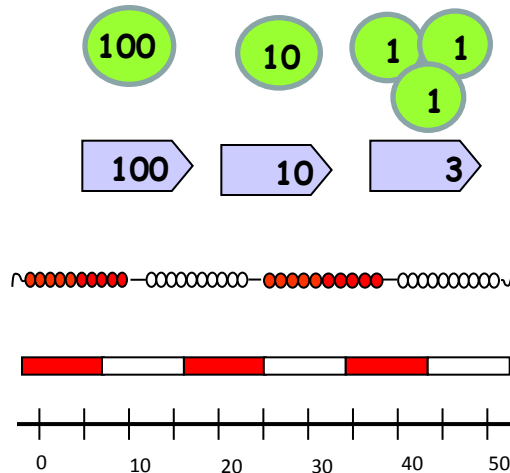


Bar Model



Year 4

Subtract numbers with up to four-digits
(formal written column method)
Understand subtraction as the inverse of addition
Solve two-step problems deciding upon the appropriate operations and methods and justifying choices made



Children **SHOULD** use manipulatives alongside algorithms to transition between practical and abstract

no exchange

$$\begin{array}{r} 68 - 23 \\ \hline \end{array}$$

$$\begin{array}{r} 60 \quad 8 \\ - 20 \quad 3 \\ \hline \end{array}$$

$$40 + 5 = 45$$

$$\begin{array}{r} 148 - 121 \\ \hline \end{array}$$

$$\begin{array}{r} 100 \quad 40 \quad 8 \\ - 100 \quad 20 \quad 1 \\ \hline \end{array}$$

$$0 + 20 + 7 = 27$$

with exchange

$$\begin{array}{r} 63 - 28 \\ \hline \end{array}$$

$$\begin{array}{r} 50 \quad 60 \quad 10 + 3 \\ - 20 \quad 8 \\ \hline \end{array}$$

$$30 + 5 = 35$$

Column subtraction (no exchange)

$$\begin{array}{r} 148 \\ - 121 \\ \hline 27 \end{array}$$

Column subtraction (with exchange)

$$\begin{array}{r} 723 - 317 \\ \hline \end{array}$$

$$\begin{array}{r} 1 \quad 1 \\ 7 \quad 2 \quad 3 \\ - 3 \quad 1 \quad 7 \\ \hline 4 \quad 0 \quad 6 \end{array}$$

$$\begin{array}{r} 723 - 367 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \quad 11 \quad 1 \\ 7 \quad 2 \quad 3 \\ - 3 \quad 6 \quad 7 \\ \hline 3 \quad 5 \quad 6 \end{array}$$

$$\begin{array}{r} 6 \quad 11 \quad 1 \\ \pounds 7 \quad 2 \quad 3 \\ - \pounds 3 \quad 6 \quad 7 \\ \hline \pounds 3 \quad 5 \quad 6 \end{array}$$

Count back in ones, tens and hundreds maintaining fluency through varied and frequent practice

Switch count between hundreds, tens and ones e.g 500, 400, 300, 290, 280, 270, 269, 268, 267

Mentally subtract HTU + ones, HTU + tens, HTU + hundreds

Perform mental calculations with two-digit numbers

Find ten and a hundred less than a number with up to three-digits

Count back in 6, 7, 9, 25 and 1000

Count back through zero to include negative numbers

Find 1000 less than a number

Continue to practise mental calculations with increasingly large numbers to aid fluency

Ensure children can solve calculations where zero is a place holder

Subtraction

Pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly.
Addition and subtraction should be taught together.

End of Year Expectations

Possible Concrete and Visual Representations

Teacher Modelling/ Children's Recording

Fluency

Year 5

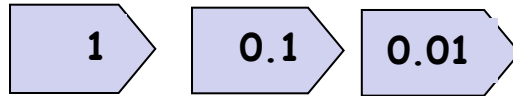
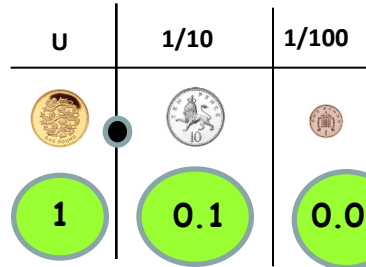
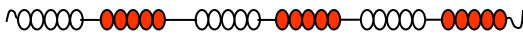
Subtract larger numbers
(formal written column method)

N.B. ENSURE CHILDREN HAVE THE OPPORTUNITY TO SUBTRACT DECIMALS WITH DIFFERING NUMBERS OF DIGITS

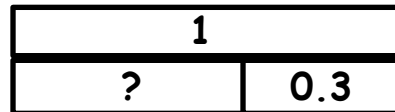
Solve multi-step problems selecting and justifying methods

Subtract numbers mentally with increasingly large numbers

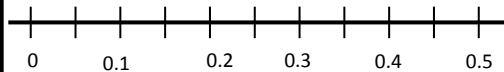
0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	2	3	4	5	6	7	8	9



Cuisenaire



Bar Model



Children might use manipulatives alongside algorithms

Column subtraction (no exchanging)

$$\begin{array}{r} 13548 \\ - 12128 \\ \hline 1420 \end{array}$$

Column subtraction
(with exchanging)

$$\begin{array}{r} \overset{2}{1} \overset{13}{3} \overset{11}{4} \overset{1}{7} 8 \\ - 12678 \\ \hline 745 \end{array}$$

Ensure children can solve calculations where zero is a place holder

$$\begin{array}{r} 1.48 \\ - 1.21 \\ \hline 0.27 \end{array}$$

Column subtraction
(no exchanging)

Column subtraction
(with exchanging)

$$\begin{array}{r} \overset{6}{7} \overset{11}{.2} \overset{1}{3} \\ - 3.67 \\ \hline 3.56 \end{array}$$

Subtraction with decimals up to three decimal places including in different contexts e.g. money and measures

Count backwards in powers of ten up to one million

Count backwards in positive and negative whole numbers through zero

Practise mental calculations with increasingly large numbers

Year 6

Subtract multi-digit numbers including numbers with up to three decimal places

(formal written column method)

ENSURE CHILDREN HAVE THE OPPORTUNITY TO SUBTRACT DECIMALS, WITH DIFFERING NUMBERS OF DIGITS

Solve multi-step problems in contexts, deciding which operations and methods to use and why

Solve more complex calculations mentally

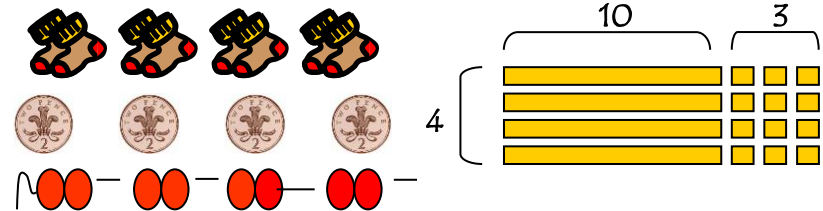
Undertake mental calculations with increasingly large numbers and more complex calculations

Structures of Multiplication (Haylock and Cockburn 2008)

Children should experience problems with all the different multiplication structures in a range of practical and relevant contexts e.g. money and measurement

Repeated addition

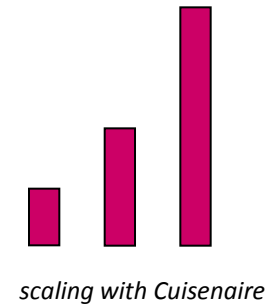
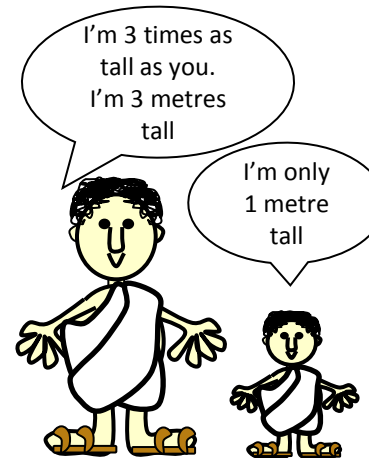
So many lots (sets) of so many
How many (how much) altogether
Per, each



Scaling

Scaling, scale factor
Doubling, trebling

So many times bigger than (longer than, heavier than, and so on)
So many times as much as (or as many as)



Commutative law

Scaling, scale factor
Doubling, trebling

So many times bigger than (longer than, heavier than, and so on)
So many times as much as (or as many as)

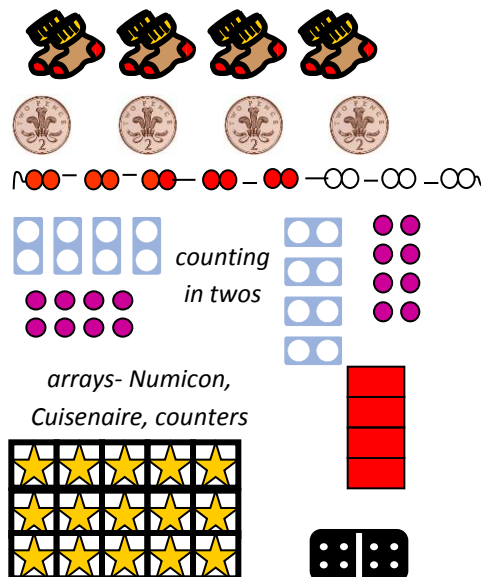
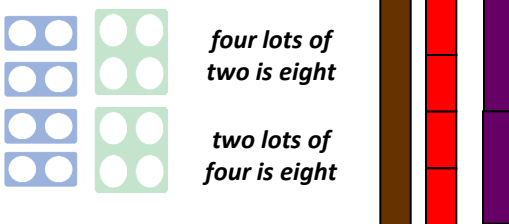
$a \times b$ and $b \times a$ are equal



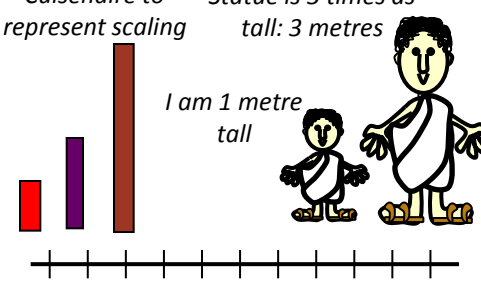
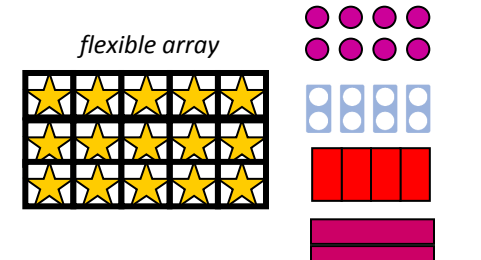
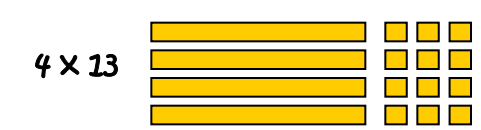
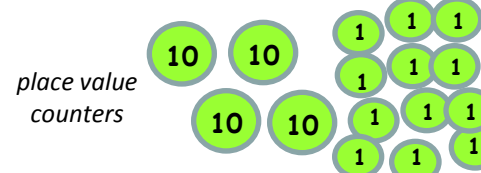
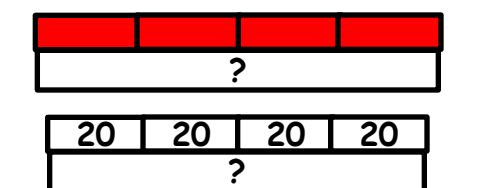
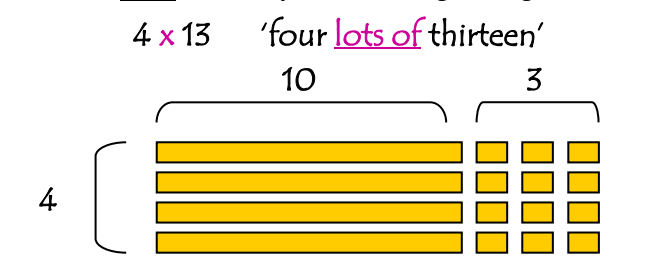
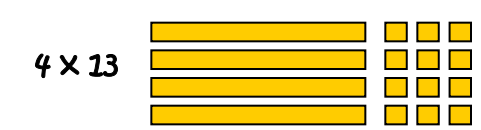
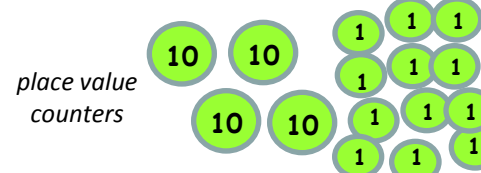
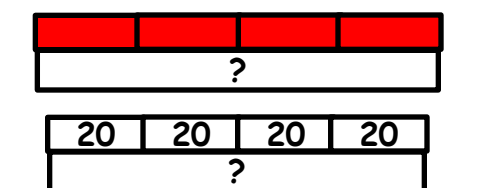
4×2 is the same as/equal to 2×4

Multiplication

Pupils develop the concept of multiplication and division and are enabled to use these operations flexibly.
 Multiplication and division should be taught together.

End of Year Expectations	Possible concrete and visual representation	Children's Recording	Fluency
<div data-bbox="19 201 193 272" style="border: 1px solid black; padding: 2px; display: inline-block;">Year 1</div> <p data-bbox="28 329 426 386">Solve single step practical problems involving multiplication</p> <p data-bbox="28 451 426 508">Use concrete objects, pictorial representations to explore grouping</p> <p data-bbox="28 558 426 644">Make connections between arrays, number patterns and counting in twos, fives and tens</p> <p data-bbox="28 722 426 751">Double numbers and quantities</p>	 <p data-bbox="484 444 755 501"><i>counting in twos</i></p> <p data-bbox="484 565 736 622"><i>arrays- Numicon, Cuisenaire, counters</i></p> <p data-bbox="484 786 678 815"><i>flexible array</i></p>	<p data-bbox="1045 215 1450 244">Practical only e.g. link to small world</p> <p data-bbox="987 265 1508 386">Using concrete objects, pictorial representations and arrays with the support of an adult – take photographs/draw pictures – if using Numicon small icons could be stuck in</p> <p data-bbox="1219 422 1354 479"><i>four lots of two is eight</i></p> <p data-bbox="1219 522 1354 579"><i>two lots of four is eight</i></p> <p data-bbox="1103 736 1335 765"><i>track with cuisenaire</i></p>	<p data-bbox="1586 287 1856 372">Count in twos, fives and tens from different multiples</p> <p data-bbox="1624 394 1837 422">e.g. 6, 8, 10, 12 etc</p> <p data-bbox="1624 444 1837 501">Emphasise number patterns</p> <p data-bbox="1624 522 1837 579">Double number and quantities</p>
<div data-bbox="19 829 193 901" style="border: 1px solid black; padding: 2px; display: inline-block;">Year 2</div> <p data-bbox="28 936 426 993">Understand multiplication as repeated addition</p> <p data-bbox="28 1022 426 1108">Calculate mathematical statements for multiplication within the tables and write them using symbols</p> <p data-bbox="28 1143 426 1200">Understand and solve problems involving arrays</p> <p data-bbox="28 1222 426 1308">Ensure children understand that multiplication is commutative (can be done in any order)</p> <p data-bbox="28 1336 426 1393">Understand that multiplication and division are inverse operations</p>	 <p data-bbox="484 986 909 1015">$2 + 2 + 2 + 2 = 4 \times 2$</p> <p data-bbox="484 1029 929 1086"><i>two add two add two add two add two = four lots of two</i></p> <p data-bbox="484 1322 774 1350"><i>flexible array</i></p>	<p data-bbox="1045 843 1508 872">Record practical work as number sentences</p> <p data-bbox="1354 936 1470 965">$4 \times 2 = 8$</p> <p data-bbox="1354 993 1470 1022">$2 \times 4 = 8$</p>	<p data-bbox="1586 901 1895 986">Count in twos, threes, fives from zero and tens from any number</p> <p data-bbox="1624 1008 1856 1036">e.g. 6, 8, 10, 12 etc</p> <p data-bbox="1586 1058 1895 1086">Emphasise number patterns</p> <p data-bbox="1586 1115 1895 1229">Introduction to multiplication tables. Practise to become fluent in multiplication facts for 2, 5 and 10</p> <p data-bbox="1586 1258 1895 1315">Solve multiplication problems mentally</p>

Multiplication – multiplication and division should be taught together– refer to structures of multiplication

End of Year Expectations	Possible concrete and visual representation	Teacher Modelling/Children's Recording	Fluency																																				
<p>Year 3</p> <p>Develop reliable written methods</p> <p>Understand and solve scaling problems</p> <p>Solve problems involving multiplication including correspondence</p>	<p><i>Cuisenaire to represent scaling</i></p> <p><i>Statue is 3 times as tall: 3 metres</i></p> <p><i>I am 1 metre tall</i></p>  <p><i>flexible array</i></p>  <p><i>arrays</i></p>  <p><i>place value counters</i></p>  <p><i>bar models</i></p> 	<p><i>Children <u>must</u> use manipulatives alongside algorithms</i></p> <p>4×13 'four <i>lots of</i> thirteen'</p>  <table border="1" data-bbox="985 485 1487 614"> <tr> <td></td> <td>10</td> <td>3</td> </tr> <tr> <td>4</td> <td>40</td> <td>12</td> </tr> </table> <p><i>Expanded methods – grid and area</i></p> <table border="1" data-bbox="985 628 1487 842"> <tr> <td>x</td> <td>10</td> <td>3</td> </tr> <tr> <td>4</td> <td>40</td> <td>12</td> </tr> </table> <p>$40 + 12 = 52$</p>		10	3	4	40	12	x	10	3	4	40	12	<p>Count from 0 in multiples of 4, 8, 50 and 100</p> <p>Use multiples of 2, 3, 4, 5, 8, 10, 50 and 100</p> <p>Practise mental recall of multiplication tables – 3, 4 and 8x times tables</p> <p>Connect the 2, 4 and 8 times tables using doubling</p> <p>Develop efficient mental methods using commutativity and multiplication facts to derive related facts e.g. $4 \times 4 \times 12 = 12 \times 4 \times 5 = 12 \times 20$</p>																								
	10	3																																					
4	40	12																																					
x	10	3																																					
4	40	12																																					
<p>Year 4</p> <p>Multiplying three numbers</p> <p>Solve two-step problems</p> <p>Multiplying by 0 and by 1</p> <p>Develop fluency in short multiplication using formal written layout</p> <p>Solve problems involving multiplication including using the distributive law, integer scaling problems and harder correspondence problems</p>	<p>4×13</p>  <p><i>place value counters</i></p>  <p><i>bar models</i></p> 	<p><i>Progressing to developing fluency in short multiplication</i></p> <table border="1" data-bbox="908 971 1487 1256"> <tr> <td>1</td> <td>3</td> <td></td> <td>1</td> <td>3</td> <td>3</td> </tr> <tr> <td>x</td> <td>4</td> <td></td> <td>x</td> <td>4</td> <td></td> </tr> <tr> <td colspan="3"><hr/></td> <td colspan="3"><hr/></td> </tr> <tr> <td>5</td> <td>2</td> <td></td> <td>5</td> <td>3</td> <td>2</td> </tr> <tr> <td colspan="3"><hr/></td> <td colspan="3"><hr/></td> </tr> <tr> <td>1</td> <td></td> <td></td> <td>1</td> <td>1</td> <td></td> </tr> </table> <p><i>Start with digits that are below five so children can practise method without encountering difficulty with multiplication tables</i></p>	1	3		1	3	3	x	4		x	4		<hr/>			<hr/>			5	2		5	3	2	<hr/>			<hr/>			1			1	1		<p>Count in multiples of 6, 7, 9, 25 and 1000</p> <p>Recall and use multiplication facts up to 12×12 with increasing fluency</p> <p>Derive multiplication facts with up to three-digits</p> <p>Recognise and use factor pairs and commutativity in mental calculations</p> <p>Use the distributive law</p> <p>Combine knowledge of number facts and rules of arithmetic to solve mental and written calculations e.g. $2 \times 6 \times 5 = 10 \times 6$</p>
1	3		1	3	3																																		
x	4		x	4																																			
<hr/>			<hr/>																																				
5	2		5	3	2																																		
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1			1	1																																			

Multiplication - multiplication and division should be taught together– refer to structures of multiplication

End of Year Expectations

Year 5

Multiply decimals with up to three decimal places

Identify multiples and factors including finding all factor pairs of a number, and common factors of two numbers

Solve problems involving all four operations where larger numbers are used by decomposing them into their factors

Multiply whole numbers and those involving decimals by 10, 100 & 1000

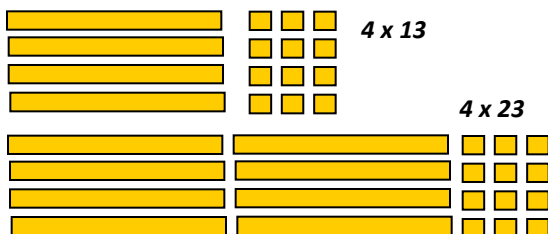
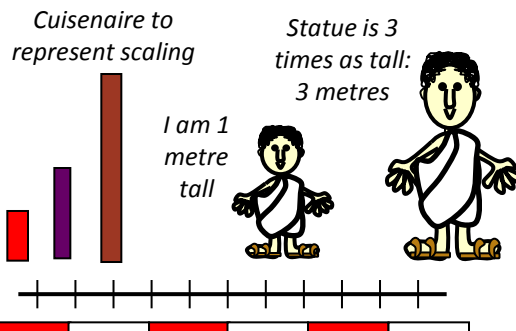
Understand and use multiplication and division as inverses including in problems involving missing numbers and balancing equations

Solve problems involving multiplication and division including scaling by simple fractions

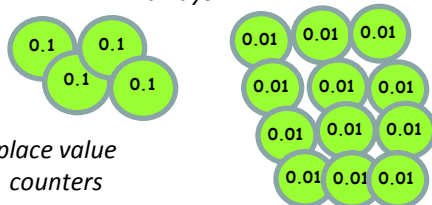
Know and use the vocabulary of prime numbers, prime factors and composite (non-prime)

Recognise and square and cube numbers and associated notation

Possible concrete and visual representation

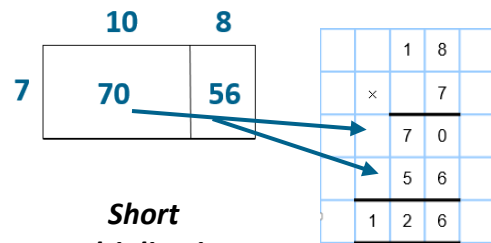


arrays



Teacher Modelling/Children's Recording

Children might use manipulatives alongside algorithms



Short multiplication

$$\begin{array}{r} 1 \ 3 \ 2 \ 4 \\ \times 6 \\ \hline \end{array}$$

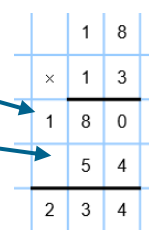
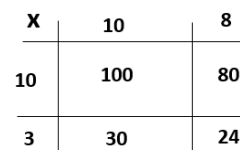
$$\begin{array}{r} 3. \ 2 \ 4 \\ \times 6 \\ \hline \end{array}$$

$$\begin{array}{r} 7 \ 9 \ 4 \ 4 \\ \times 6 \\ \hline \end{array}$$

$$\begin{array}{r} 1 \ 9. \ 4 \ 4 \\ \times 6 \\ \hline \end{array}$$

$$\begin{array}{r} 1 \ 1 \ 2 \\ \times 6 \\ \hline \end{array}$$

$$\begin{array}{r} 1 \ 2 \\ \times 6 \\ \hline \end{array}$$



Long multiplication

$$\begin{array}{r} 1 \ 3 \ 2 \ 4 \\ \times 2 \ 6 \\ \hline \end{array}$$

$$\begin{array}{r} 3. \ 2 \ 4 \\ \times 2 \ 6 \\ \hline \end{array}$$

$$\begin{array}{r} 7 \ 9 \ 4 \ 4 \\ \times 2 \ 6 \\ \hline \end{array}$$

$$\begin{array}{r} 1 \ 9. \ 4 \ 4 \\ \times 2 \ 6 \\ \hline \end{array}$$

$$\begin{array}{r} 2 \ 6 \ 4 \ 8 \ 0 \\ \times 1 \ 1 \ 2 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \ 4. \ 8 \ 0 \\ \times 1 \ 2 \\ \hline \end{array}$$

$$\begin{array}{r} 3 \ 4 \ 4 \ 2 \ 4 \\ \times 1 \ 1 \ 1 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \ 4. \ 2 \ 4 \\ \times 1 \ 1 \\ \hline \end{array}$$

Fluency

Count forwards in steps of powers of 10 from any given number up to 1 000 000

Practise and extend use of formal written method of short multiplication

Apply all multiplication tables frequently. Commit them to memory and use them confidently to make larger calculations

Multiply numbers mentally drawing upon known facts

Year 6

Multiply numbers up to 4-digit x TU

Multiply numbers with up to two decimal places x whole number

Multiply multi-digit numbers up to four-digits by a two-digit whole number

Multiply single-digit numbers with up to two-decimal places by whole numbers

Solve problems involving all four operations

Undertake mental calculations with increasingly large numbers

Continue to use all multiplication tables to calculate mathematical statements in order to maintain fluency

Structures for Division (Haylock and Cockburn 2008)

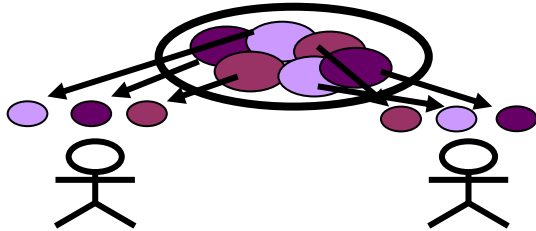
Children should experience problems with the different division structures in a range of practical and relevant contexts e.g. money and measurement

Equal-sharing

Sharing equally between
How many (much) each?

6 shared equally by 2

$$6 \div 2$$

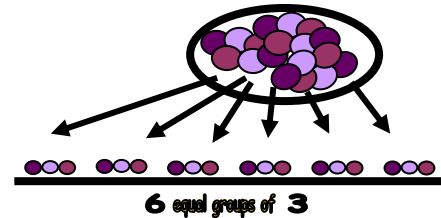


Inverse of multiplication (Grouping)

So many lots (sets/groups) of so many
Share equally in to groups of ...

$$18 \div 3$$

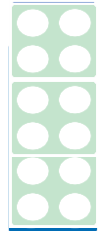
18 divided into
equal groups of 3s



Divide twelve into equal
groups of four



Make 12



Overlay
groups of
four

= 3

Ratio structure

comparison

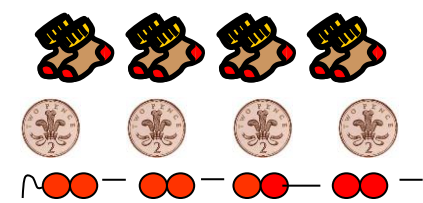

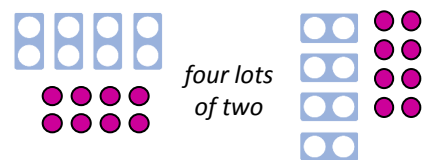
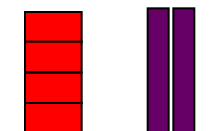


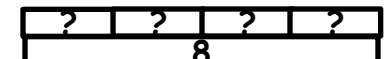
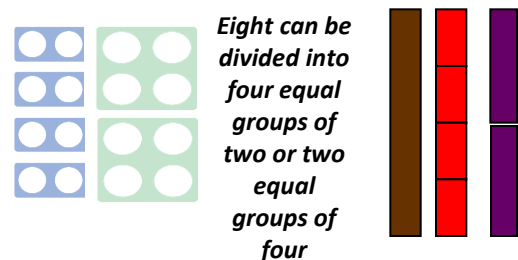

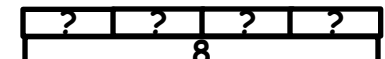
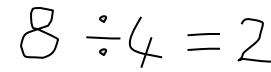
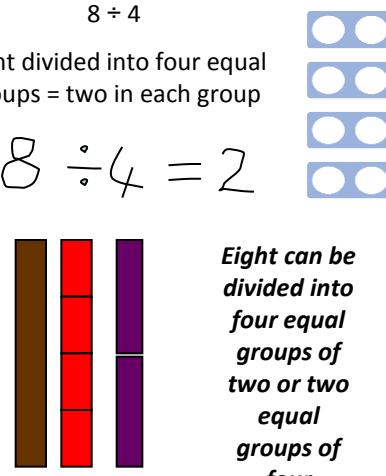
inverse of scaling structure of multiplication
scale factor (decrease)

Barney earns three times more than Fred. If
Barney earns £900 how much does Fred earn?

Jo's journey to school is three times as
long as Ella's. If Jo walks to school in
30 minutes how long does it take Ella?

Division

Pupils develop the concept of multiplication and division and are enabled to use these operations flexibly.
Multiplication and division should be taught together.

End of Year Expectations	Possible concrete and visual representation	Teacher Modelling/Children's Recording	Fluency
<p>Year 1</p> <p>Solve single step practical problems involving division</p> <p>Use concrete objects, pictorial representations</p> <p>Understand division as grouping and sharing</p> <p>Use the language of 'sharing equally between'</p>	<p><i>counting in groups of twos</i></p>  <p><i>straw bundles</i></p>   <p><i>Numicon and counter arrays</i></p>  <p><i>Cuisenaire</i></p> <p><i>doubling</i></p>  <p><i>flexible array</i></p>  <p><i>bar models</i></p> 	<p>Practical only e.g. link to small world</p> <p>Using concrete objects, pictorial representations and arrays with the support of an adult – take photographs/draw pictures – if using Numicon small icons could be stuck in</p>  <p><i>Eight can be divided into four equal groups of two or two equal groups of four</i></p>	<p>Count in twos, fives and tens from different multiples</p> <p>e.g. 6, 8, 10, 12 etc</p> <p>Emphasise patterns</p> <p>Find simple fractions eg half and quarter, of objects, numbers and quantities</p>
<p>Year 2</p> <p>Solve single step practical problems involving division</p> <p>Use concrete objects, pictorial representations</p> <p>Understand division as grouping</p> <p>Find halves and then quarters</p> <p>Work with a range of materials and contexts in which multiplication and division relate to grouping and sharing discrete quantities and to arrays</p>	<p><i>four lots of two</i></p> <p><i>two lots of four</i></p> <p><i>doubling</i></p> <p><i>flexible array</i></p>  <p><i>bar models</i></p> 	<p>Record as number sentences using \div and $=$</p> <p>$8 \div 4$</p> <p>Eight divided into four equal groups = two in each group</p>   <p><i>Eight can be divided into four equal groups of two or two equal groups of four</i></p>	<p>Count back in twos, threes, fives from zero and tens from any number</p> <p>e.g. 12, 10, 8, 6 etc</p> <p>Emphasise patterns</p> <p>Connect ten times table to place value and five times table to divisions on a clock face</p> <p>Introduction to multiplication tables. Practise to become fluent in division facts for 2, 5 and 10</p> <p>Solve division problems involving grouping and sharing</p>

Division - multiplication and division should be taught together– refer to structures of division

End of Year Expectations

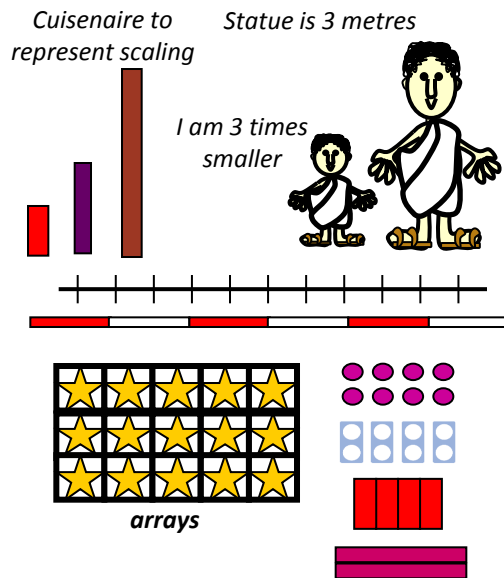
Possible concrete and visual representation

Teacher Modelling/Children's Recording

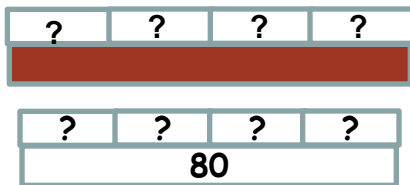
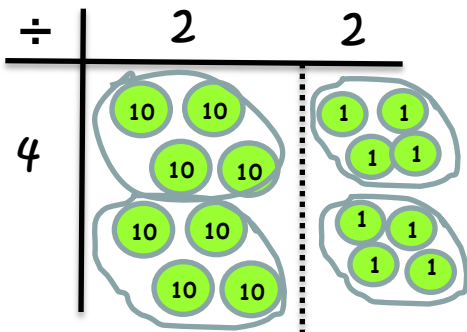
Fluency

Year 3

- Develop a reliable written method for division
- Solve problems involving missing numbers
- Solve problems including those that involve scaling
- Recognise, find and name $\frac{1}{2}$ and $\frac{1}{4}$ of an object, shape or quantity
- Understand the link between unit fractions and division
- Connect $1/10$ to division by 10
- Count in tenths



$$88 \div 4$$



bar models

Children should use manipulatives alongside algorithms



Repeated subtraction - chunking

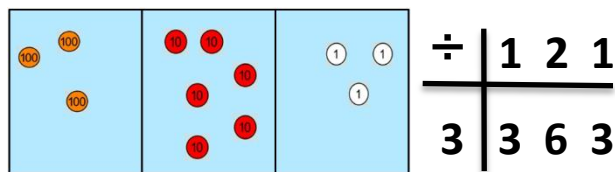
Ensure children see/understand the link between grouping on a number line and vertical recording for chunking

$$95 \div 5 = 19$$

$$\begin{array}{r} 95 \\ - 50 \quad (\underline{10} \times 5) \\ \hline 45 \\ - 25 \quad (\underline{5} \times 5) \\ \hline 20 \\ - 20 \quad (\underline{4} \times 5) \\ \hline 0 \end{array}$$

Fact Box

$$\begin{array}{l} 2 \times 5 = 10 \\ 5 \times 5 = 25 \\ 10 \times 5 = 50 \end{array}$$



$$321 \div 3$$

Short division-
no remainders

$$560 \div 4$$

$$\begin{array}{r} \div \quad 1 \ 4 \ 0 \\ \hline 4 \quad 5 \ 6 \ 0 \end{array}$$

Recall and use related division facts for the 3, 4 and 8x tables (Continue to practise other tables)

Write and calculate mathematical statements for division using what is known

Use division facts to derive related division facts e.g. using $6 \div 3 = 2$ to work out $60 \div 3 = 20$

Continue to practise recalling division facts for multiplication tables up to 12×12

Practise mental methods and extend this to three-digit numbers for example $200 \times 3 = 600$ into $600 \div 3 = 200$

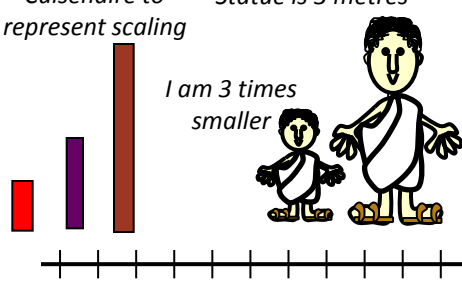

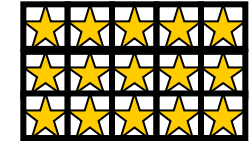
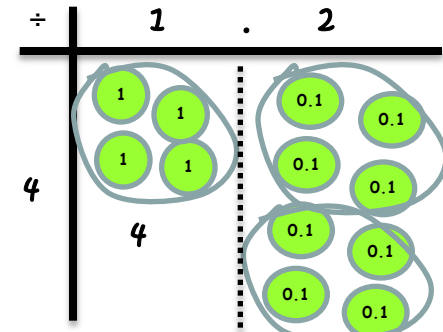
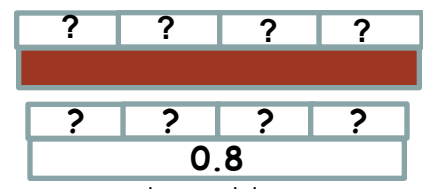
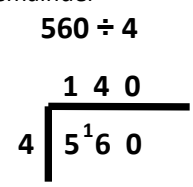
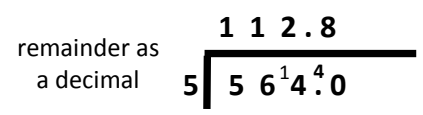
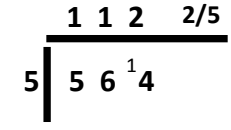
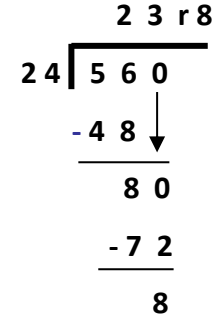
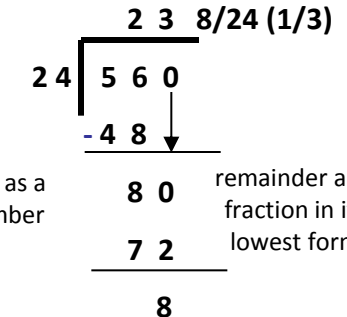
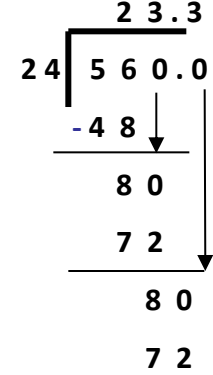
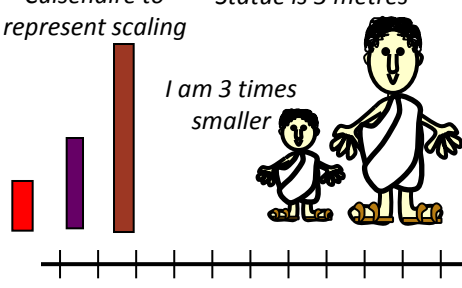

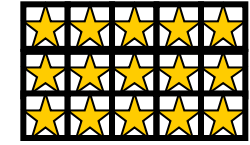
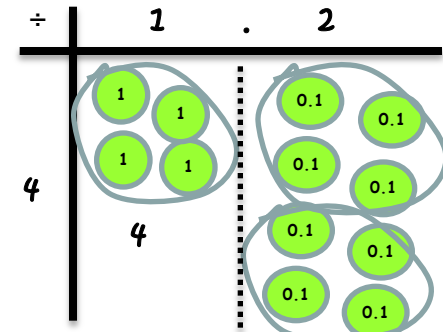
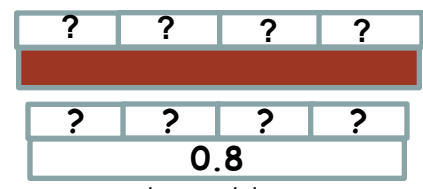
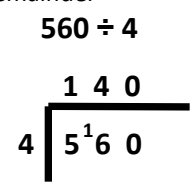
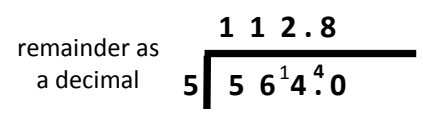
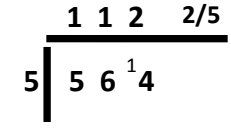
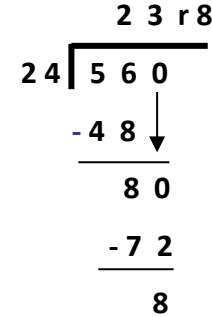
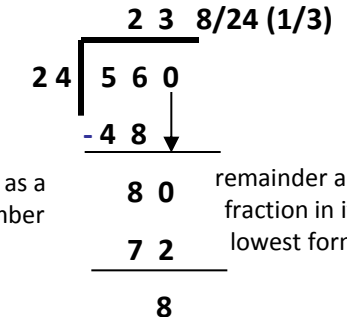
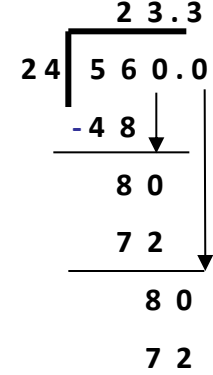
Use place value, known and derived facts to divide mentally, including dividing by 1

Recognise and use factor pairs and commutativity in mental calculations

Year 4

- Become fluent in the formal written method of short division with exact answers when dividing by a one-digit number
- Divide one- or two-digit numbers by 10 or 100, identifying value of digits as tenths or hundredths
- Solve two-step problems in different contexts, choosing the appropriate operation, working with increasingly harder numbers including correspondence questions e.g. three cakes shared equally between 10 children

Division - multiplication and division should be taught together– refer to structures of division

End of Year Expectations	Possible concrete and visual representation	Teacher Modelling/Children's Recording	Fluency
<p>Year 5</p> <p>Identify factors, including finding all factor pairs of a number, and common factors of two numbers</p> <p>Practise and extend the formal written method of short division: numbers up to four-digits by a one-digit number</p> <p>Interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding as appropriate for the context</p> <p>Use multiplication and division as inverses</p> <p>Solve problems involving division including scaling down</p> <p>Divide whole numbers and those involving decimals by 10, 100 & 1000</p>	<p><i>Cuisenaire to represent scaling</i></p>  <p><i>Statue is 3 metres</i></p>  <p><i>flexible arrays</i></p>  <p>$4.8 \div 4$</p>   <p><i>bar models</i></p>	<p><i>Children might use manipulatives alongside algorithms</i></p> <p>without remainder</p> $560 \div 4$  <p>remainder as a decimal</p> $564 \div 5$  <p>remainder as a fraction</p> $564 \div 5$  <p>long division</p> $560 \div 24$   	<p>Count backwards in steps of powers of 10 for any given number up to 1 000 000</p> <p>Count backwards with positive/negative whole numbers through zero</p> <p>Practise mental calculation with increasingly large numbers</p> <p>Apply all multiplication tables and related division facts frequently, commit them to memory and use them to confidently to make larger calculations</p>
<p>Year 6</p> <p>Divide numbers up to 4-digits by a 2-digit whole number using formal written methods of long division, interpret remainders as whole numbers, fractions or by rounding, as appropriate for the context</p> <p>Divide numbers with up to 2 decimal places by 1-digit and 2-digit whole numbers, initially in practical contexts involving money and measures</p> <p>Understand the relationship between unit fractions and division</p> <p>Recognise division calculations as the inverse of multiplication</p> <p>Solve problems involving division</p>	<p><i>flexible arrays</i></p>  <p><i>Statue is 3 metres</i></p>  <p><i>flexible arrays</i></p>  <p>$4.8 \div 4$</p>   <p><i>bar models</i></p>	<p><i>Children might use manipulatives alongside algorithms</i></p> <p>without remainder</p> $560 \div 4$  <p>remainder as a decimal</p> $564 \div 5$  <p>remainder as a fraction</p> $564 \div 5$  <p>long division</p> $560 \div 24$   	<p>Practise division for larger numbers, using the formal written methods of short and long division</p> <p>Continue to use all multiplication tables and division facts to maintain fluency</p> <p>Perform mental calculations, including with mixed operations and larger numbers</p>